TRANSPORT PROCESSES IN A MULTI-COMPONENT ASSEMBLY ON THE BASIS OF GENERALIZED BGK COLLISION MODEL

BY C. DEVANATHAN, (MISS) C. UBEROI AND P. L. BHATNAGAR
(Department of Applied Mathematics, Indian Institute of Science, Bangalore-12)

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ABSTRACT

Bhatnagar, Gross and Krook developed a collision model for one component neutral assembly of particles in order to overcome the inherent difficulties of the Boltzmann collision integral. This has been generalized to an N-component assembly of charged and neutral particles by Bhatnagar and Devanathan. However, the transport equations obtained directly from these kinetic equations are far from simple. In this paper, simpler and elegant transport equations have been obtained by expanding the distribution functions in generalized Hermite Polynomials following Grad. From these generalized coefficients of direct electrical conductivity, diffusivity, viscosity, and heat conductivity are obtained in the presence of magnetic field. Also the relaxation times have been calculated. These naturally lead to a mechanism of the occurrence of Gross-gaps.

1. INTRODUCTION

The transport processes are essentially non-equilibrium processes. In the study of non-equilibrium processes one attempts to derive from the kinetic equations a consistent closed system of transport equations involving the macroscopic quantities associated with the system like density, velocity, temperature, stresses, heat flux, etc. In such macroscopic equations certain parameters occur. For instance, the stresses are proportional to certain space derivatives of velocity components. The corresponding coefficient of proportionality is defined as the coefficient of viscosity. Similarly, the heat flux vector is directly proportional to the temperature gradient and the coefficient of proportionality is the coefficient of heat conductivity and so on. In certain simple flow problems of an ideal gas, we can identify these coefficients exactly as the momentum transfer and heat transfer per unit area per unit time due to molecular interactions. But in general, the dependence is very complicated and we consider the former statement as the definition of transport coefficients. Thus, the main purpose of the present paper is to start with suitable kinetic equations and to deduce a closed system of transport equations in order to obtain expressions for transport coefficients such as viscosity, heat conductivity and electrical conductivity. In § 2, we shall discuss the basic kinetic equations.
which we have used and in § 3, the outline of the procedure for solution is explained and the closed system of transport equations are derived. In § 4, we consider three simple problems to derive the coefficients of viscosity, heat conductivity and the electrical conductivity. These simple processes are generalized and stationary non-equilibrium processes in the presence of magnetic field are considered in § 5. Finally, in § 6, we discuss the unsteady relaxation problem and attempt a plausible physical explanation of Gross gaps in frequency spectrum.

2. Kinetic Equations

Consider an assembly of \( N \) kinds of particles. Let \( m_s \) and \( e_s \) denote the mass and charge of a particle of \( s \)-type. Further, let, at time \( t \) and position \( r \), \( \dot{\xi}_s \) be the molecular velocity and \( F_s \) be the external non-electromagnetic force acting on that particle. Then, the state of the system is described by the distribution functions \( f_s(\xi_s, r, t) \) satisfying the Maxwell-Boltzmann equations and the self-consistent electromagnetic equations. With the usual notation of Chapman and Cowling, these equations are:

Maxwell-Boltzmann equations

\[
\frac{\partial f_s}{\partial t} + \dot{\xi}_s \frac{\partial f_s}{\partial x_i} + \frac{1}{m_s} \left[ F_{si} + e_s (E_i + \left[ 1/c \right] \epsilon_{ijk} \dot{\xi}_{sj} H_k) \right] \frac{\partial f_s}{\partial \dot{\xi}_{si}} = \sum_{j=1}^{N} \int \int \int \left[ f_j'(\xi_{sj}, r, t) f_{sj}(\xi_{sj}, r, t) - f_j(\xi_{sj}, r, t) f_{sj}(\xi_{sj}, r, t) \right] \times \nonumber
\]
\[\times g_{js} bdb d\epsilon d\dot{\xi}_{sj}, \quad s = 1, \ldots, N \quad [2.1]\]

Maxwell equations

\[
c \nabla \times H = 4\pi J + \partial E/\partial t, \quad [2.2]
\]
\[
c \nabla \times E = -\partial H/\partial t, \quad [2.3]
\]
\[
\nabla \cdot H = 0, \quad [2.4]
\]
\[
\nabla \cdot E = 4\pi q, \quad [2.5]
\]

where the current density \( J \) and the charge density \( q \) are given by

\[
J = \sum_{j=1}^{N} e_j \int f_j(\xi_{sj}, r, t) d\xi_{sj}, \quad [2.6]
\]

and

\[
q = \sum_{j=1}^{N} e_j \int f_j(\xi_{sj}, r, t) d\xi_{sj}. \quad [2.7]
\]
The above equations are highly coupled nonlinear integro-partial differential equations and in order to simplify the basic kinetic equations, Bhatnagar, Gross and Krook\(^2\) proposed a simple tractable Collision model retaining the essential physical characteristics of the Maxwell-Boltzmann equations. This has been generalized to a multicomponent assembly by Bhatnagar and Devanathan\(^3\). In the collision integral, the term

\[-f_s(\xi_s, r, t) \int f_j(\xi_j, r, t) g_{js} b d\varepsilon d\xi_j\]

represents the number of \(s\)-th type of particles removed from the velocity range \((\xi_s, d\xi_s)\) by the interaction with \(j\)-th type of particles and hence replaced by an equivalent model

\[-N_j(r, t) \frac{f_s(\xi_s, r, t)}{\sigma_{js}}\]

where \(N_j(r, t)\) is the number density of \(j\)-th type of particles given by

\[N_j(r, t) = \int f_j(\xi_j, r, t) d\xi_j.\]

The nonlinear term

\[\int \int f_j^\prime(\xi_j^\prime, r, t) f_s^\prime(\xi_s^\prime, r, t) g_{js} b d\varepsilon d\xi_j\]

representing the number of particles brought into the velocity range concerned is replaced by the following:

\[(\text{total number of collisions per unit volume per unit time}) \times \times (\text{probability that the particle goes into the concerned velocity range})\]

We shall denote the average total number of collisions between \(s\)-th type and \(j\)-th type by

\[\frac{N_j N_s}{\sigma_{js}} = \int \int f_j(\xi_j, r, t) f_s(\xi_s, r, t) g_{js} b d\varepsilon d\xi_j d\xi_s; \quad [2.8]\]

and instead of taking the detailed mechanism of collision in evaluating the probability mentioned above, we take it to be locally Maxwellian given by

\[\Phi_{js}(\xi_s, r, t) = \left(\frac{m_s}{2\pi K T_{js}}\right)^{3/2} \exp \left[-\frac{m_s}{2 K T_{js}} (\xi_s - u_{js})^2\right]\]

assuming that the \(s\)-particles are scattered randomly by the \(j\)-particles. In the above expression \(K\) is the Boltzmann constant, \(u_{js}\) and \(T_{js}\) are mean velocity and temperature of the scattered \(s\)-th type of particles during their interaction.
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with \( j \)-th type of particles. For these cross-velocities and temperatures we choose the phenomenological relations

\[
\begin{align*}
\mathbf{u}_{js} &= a_{jj} \mathbf{u}_{jj} + a_{js} \mathbf{u}_{ss}, \\
T_{is} &= \tilde{b}_{jj} T_{jj} + \tilde{b}_{js} T_{ss} + D_{js} \mathbf{u}_{jj}^2 + E_{js} \mathbf{u}_{jj} \cdot \mathbf{u}_{ss} + F_{js} \mathbf{u}_{ss}^2,
\end{align*}
\]

where \( \mathbf{u}_{jj} \) and \( T_{jj} \) are the mean velocity and temperature of the \( i \)-th type of particles. Subjecting the collision terms to Maxwell's relaxation problem and to the instantaneous conservation laws of mass, momentum and energy, we get (for complete details refer\(^3\))

\[
\begin{align*}
a_{jj} &= 1 - a_{js}, \\
D_{js} &= -\frac{1}{2} E_{js} = F_{js}, \\
\tilde{b}_{jj} &= 1 - \tilde{b}_{js}, \\
m_s a_{jj} = m_j a_{ss} &= A_{js} = A_{sj}, \\
\tilde{b}_{jj} &= \tilde{b}_{ss} = B_{js} = B_{sj},
\end{align*}
\]

and

\[
D_{js} + D_{sj} = 1/(3k) A_{js} (2 - a_{jj} - a_{ss}).
\]

These relations determine only half of the phenomenological constants. But, considering the average momentum transfer and energy transfers, we find

\[
A_{js} = \frac{m_j m_s}{m_j + m_s} \left[ \frac{m_s}{m_j m_s/(m_j + m_s)} \right]_{av}
\]

and

\[
6K D_{js} = \frac{m_j m_s}{m_j + m_s} \left[ \frac{1}{2} m_s \left( \frac{m_j}{m_j m_s/(m_j + m_s)} \right)^2 \right]_{av}
\]

Also \( B_{js} \) is just the coefficient of direct heat transfer between the two components. From the knowledge of the law of interaction, these constants have been determined in reference\(^3\). For ready reference we shall record them below:

\[
A_{js} = \frac{m_j m_s}{m_j + m_s} \quad \text{for elastic collisions}
\]

\[
\approx 0.113 \frac{m_j m_s}{m_j + m_s} \quad \text{for Coulomb law}
\]

\[
\approx 0.023 \frac{m_j m_s}{m_j + m_s} \quad \text{for Maxwellian law}
\]
Thus, according to the above model, we replace the set of Maxwell-Boltzmann equations [2.1] by the following set of kinetic equations:

\[
\frac{\partial f_s}{\partial t} + \xi_{sl} \frac{\partial f_s}{\partial x_l} + \frac{1}{m_s} \left[ F_{sl} + e_s \left( E_i + \frac{1}{c} \epsilon_{ijk} \xi_{sj} H_k \right) \frac{\partial f_s}{\partial \xi_{sl}} \right] = \sum_{j=1}^{N} \frac{N_j}{\sigma_{js}} \left( -f_s + N_j \Phi_{js} \right),
\]

for Maxwellian law.

This leads to physically meaningful transport equations. However, as in the earlier transport equations of Chapman or Burnett, this set of transport equations also does not form a closed system of equations.

3. TRANSPORT EQUATIONS FOR NON-EQUILIBRIUM PHENOMENA

We have already pointed out that the set of transport equations obtained earlier do not form a closed set. The usual procedure to obtain transport equations, followed often, is to consider the given system in a known equilibrium state specified by the distribution functions \( f_{se} \) and a small deviations \( f_{sl} \) from this equilibrium state resulting from the preassigned non-equilibrium situations like density, velocity, and temperature gradients. Then using the perturbation techniques first order transport equations are established which yield directly the respective transport coefficients.

One of the earliest of such methods is the classical solution of Chapman-Enskog-Hilbert. We may note that their solution turns out to be a series solution in terms of a parameter involving the mean free path and even the first order corrections are quite complicated and higher approximations are almost prohibitive owing to enormous mathematical complexity.

Another effective method, due to Lorentz and adopted successfully by Morse et al. and Margenau, is to expand the velocity dependence of the distribution function in spherical harmonics in velocity space. Spherical harmonics in velocity space are eigenfunctions of Boltzmann collision operator for Lorentzian gas, the corresponding eigenvalues depending upon the collision
frequency. Naze\textsuperscript{12} extended these results to more general case. Thus, the mathematical advantage of the method is off-set by the fact that physically this expansion is a series expansion in terms of collision frequency and has very limited scope.

Grad\textsuperscript{13} developed another method, which is also an orthogonal function expansion in the velocity space, employing the generalized Hermite Polynomials\textsuperscript{14}. This method has decisive advantage over the earlier methods. The distribution functions are taken in the form

\[ f_s = f_{so} \left[ \sum a^{(n)} H^{(n)} \right], \quad s = 1, 2, ..., N, \]  

where the weighting factors \( f_{so} \) are exactly the equilibrium distributions. The second and the successive terms represent the deviation from the postulated equilibrium state with coefficients \( a^{(n)} \) as linear combinations of the macroscopic variables of the system. Such a procedure is evidently very much suited for the solution of transport processes under consideration. Besides these expansions may be managed to be convergent by taking the deviations to be small. Further, on truncating the series at a convenient stage, we can obtain a closed system of equations for the physical variables.

Thus, the kinetic equations [2.9] and the expansions [3.1] form the basis of the present investigation.

In the subsequent working we concentrate on a three component assembly consisting of electrons, ions and neutral particles respectively denoted by the suffixes \( \alpha, \beta \) and \( \gamma \), as the generalization to any number of components is straightforward. Further, in order to facilitate the orthogonal function expansion, we introduce the nondimensional distribution functions \( g_a \) of the nondimensional molecular velocities \( v_a \) defined by

\[ g_a = \frac{1}{N_a} \left( \frac{K T_{aa}}{m_a} \right)^{3/2} f_a \left( \frac{v_a}{\xi_a}, r, t \right), \]  

\[ v_a = \left( \frac{m_a}{K T_{aa}} \right)^{1/2} \xi_a. \]

Then, the general expansion can be written in the form

\[ g_a \left( v_a, r, t \right) = \omega \left( v_a \right) \sum_{n=0}^{\infty} a^{(n)}_a \left( r, t \right) H^{(n)} \left( v_a \right), \]  

where

\[ \omega \left( v_a \right) = \frac{1}{(2\pi)^{3/2}} \exp \left[ -\frac{1}{2} v_a^2 \right]. \]

the nondimensional Maxwellian distribution function corresponding to the postulated equilibrium situation. Because of the orthogonality property of the Hermite polynomials with kernel \( \omega_a \), we have

\[ a^{(n)}_a \left( r, t \right) = \frac{1}{X^{(n)}} \int H^{(n)} \left( v_a \right) g_a \left( v_a, r, t \right) d v_a, \]
where

\[ X^{(n)} = \int (s^I (v_a) [H^{(n)} (v_a)])^2 dv_a. \]  \[ \text{[3.7]} \]

Since \( H^{(n)} (v_a) \) is merely a polynomial in velocity components, the above expression clearly shows that \( a^{(n)}_a (r, t) \) are linear combinations of the moments of the distribution function. For convenience we have given the Hermite polynomials up to the fourth degree and the corresponding coefficients in terms of the moments of \( g_a \) in Appendix 2. Since we are dealing with Hermite polynomials in three dimensional velocity space, the number of distinct types of \( a^{(n)} \) of order \( n \) can be shown to be (Appendix 3),

\[ \frac{1}{72} \left[ 47 + 36 n + 6 n^2 + ( -1)^n \left( 9 + 16 \cos \frac{n \pi}{3} \right) \right] \]  \[ \text{[3.8]} \]

This differs from Grad’s results who, from an analogy with Cartesian tensors, inferred that there are \( n! \) distinct components. This leads to a slight inaccuracy in his numerical coefficients.

Accordingly, the non-dimensional distribution functions \( g_a \) satisfy the kinetic equation

\[
\frac{\partial g_a}{\partial t} + \left( \frac{K T}{m_a} \right)^{1/2} v_{aI} \frac{\partial g_a}{\partial x_I} + \frac{1}{m_a} \left[ F_{aI} + e_a (E_t + \frac{1}{c} \left( \frac{K T}{m_a} \right)^{1/2} \epsilon_{ijk} v_{aI} H_k) \right] \frac{\partial g_a}{\partial x_I} \\
+ g_a \left[ \frac{\partial}{\partial t} \left( \log N_a \left( \frac{m_a}{K T} \right)^{3/2} \right) + \left( \frac{K T}{m_a} \right)^{1/2} v_{aI} \frac{\partial}{\partial x_I} \left( \log N_a \frac{m_a}{K T} \right)^{3/2} \right] \\
= - \tau_a g_a + \sum_{\delta = \alpha} N_\delta \sigma_{\delta a} \left( \frac{T_{aa}}{2 \pi T_{\delta a}} \right)^{3/2} \exp \left[ - \frac{T_{aa}}{2 T_{\delta a}} (v - v_{\delta a})^2 \right] \]  \[ \text{[3.9]} \]

where

\[ \tau_a = \frac{N_a}{\sigma_{aa}} + \frac{N_\beta}{\sigma_{\beta a}} + \frac{N_\gamma}{\sigma_{\gamma a}} \]  \[ \text{[3.10]} \]

and \( v_{aa}, v_{\beta a}, v_{\gamma a} \) are the non-dimensional mean velocities \( u_{aa}, u_{\beta a}, \) and \( u_{\gamma a}. \) Similar equations hold for \( \beta \) and \( \gamma \) components of the assembly. In [3.9] and in the subsequent calculations, we shall use the suffixes \( i, j, k, l, m, n, \ldots \) for dummy summation indices and the suffixes \( r, s, t, u, v, \ldots \) for fixed indices.

Substituting the expansion [3.4] in the equation [3.9] and integrating with respect to \( v_a \) after multiplying with \( H^{(n)} (v_a) \), we get the equation for \( a^{(n)}_a \). Since \( v_a \) is explicitly present in the equation [3.9], the equation for \( a^{(n)}_a \) will contain \( a^{(n+1)}_a \). Hence a suitable cut off is essential to obtain a closed set of equations for \( a^{(n)}_a \). To effect this cut off, we have retained only the terms up to \( a^{(4)}_a \). The explicit expressions (Appendix 2) for these coefficients in terms of the moments of the distribution function lend justification for the cut off at \( a^{(4)}_a \). For instance, \( a^{(1)}_a \) contains the mean velocities and consequently for an assembly consisting of charged particles the current density term. Thus, \( a^{(1)}_a \) takes into account the anisotropy caused in the momentum space.
Similarly, $a_{(2)}^{(2)}$ contains both the material stresses and the Maxwell stresses and the Poynting flux. $a_{(3)}^{(2)}$ mainly accounts for heat flux and energy flux and $a_{(4)}^{(4)}$ takes account of the interaction between material stresses and Maxwell stresses. This fact has been pointed out by Burgers\textsuperscript{15} and Bhatnagar\textsuperscript{16}. Hence we have included the terms up to $a_{(4)}^{(4)}$ and neglected $a_{(5)}^{(5)}$ and subsequent terms, since the above physical quantities govern most of the natural phenomena that occur. However, if any physical situation warrants the inclusion of some particular higher order term, the formalism is general enough to include it. Consequently we consider the truncated expansion.

$$g_a = \omega_a \sum_{n=0}^{4} a_{(n)} H_{(n)} (v_a). \quad [3.11]$$

This process of truncation provides a natural way of expressing the fifth and higher order moments of the distribution function in terms of moments up to fourth order in contrast to the arbitrary definitions of earlier approaches\textsuperscript{17}.

The equation for $a_{(n)}^{(n)}$ is given by:

$$X_{(n)} \frac{\partial a_{(n)}^{(n)}}{\partial t} - \left[ \frac{n+3}{2} X_{(n)} a_{(n)}^{(n)} + X_{(n-2)} \delta_{(2)}^{(2)} a_{(n-2)}^{(n-2)} \right] \frac{\partial}{\partial t} \left( \log \frac{m_a}{K T_{aa}} \right)$$

$$+ \left( \frac{K T_{aa}}{m_a} \right)^{1/2} \left( X_{(n+1)} \frac{\partial a_{(n+1)}^{(n+1)}}{\partial x_i} \right)$$

$$- \frac{n+4}{2} \left[ X_{(n+1)} a_{(n+1)}^{(n+1)} + X_{(n-1)} \delta_{(2)}^{(2)} a_{(n-1)}^{(n-1)} \right] \frac{\partial}{\partial x_i} \left( \log \frac{m_a}{K T_{aa}} \right)$$

$$+ X_{(n-1)} \delta_{(2)}^{(2)} \frac{\partial a_{(n-1)}^{(n-1)}}{\partial x_i} - X_{(n-3)} \delta_{(2)}^{(2)} a_{(n-3)}^{(n-3)} \frac{\partial}{\partial x_i} \left( \log \frac{m_a}{K T_{aa}} \right)$$

$$- \left( \frac{m_a}{K T_{aa}} \right)^{1/2} \left( \frac{e_a}{m_a} E + \frac{1}{m_a} F_{ai} \right) X_{(n-1)} \delta_{(2)}^{(2)} a_{(n-1)}^{(n-1)}$$

$$- \frac{e_a}{c m_a} \epsilon_{ijk} H_k \left[ X_{(n)} \delta_{(2)}^{(2)} a_{ai}^{(n)} + X_{(n-2)} \delta_{(2)}^{(2)} a_{ai}^{(n-2)} \right]$$

$$+ \frac{\partial}{\partial t} \left\{ \log N_a \left( \frac{m_a}{K T_{aa}} \right)^{3/2} \right\} X_{(n)} a_{(n)}^{(n)}$$

$$+ \left( \frac{K T_{aa}}{m_a} \right)^{1/2} \frac{\partial}{\partial x_i} \left\{ \log N_a \left( \frac{m_a}{K T_{aa}} \right)^{3/2} \right\} \left[ X_{(n+1)} a_{(n+1)}^{(n+1)} + X_{(n-1)} \delta_{(2)}^{(2)} a_{(n-1)}^{(n-1)} \right]$$

$$- \tau_a X_{(n)} a_{(n)}^{(n)} + \sum_{\delta} \frac{N_{\delta}}{\sigma_{\delta a}} \left( \frac{T_{aa}}{2 \pi T \delta a} \right)^{3/2} A_{(n)}^{(n)}. \quad [3.12]$$
where

$$A_{\delta a}^{(n)} = \int H^{(n)}(\omega^+ + v_{\delta a}) \exp \left[ -\frac{T_{\alpha a} \omega_0^+}{2T_{\delta a}} \right] d\omega^+.$$  \[3.13\]

Putting $a_\alpha^{(3)}$, $a_\alpha^{(4)}$, ..., equal to zero, we get a closed system of equations for $a_\alpha^{(n)}$, $(n = 0, 1, 2, 3, 4)$. Reverting back to the physical variables, we have the following transport equations:

$$\frac{\partial N_a}{\partial t} + \frac{\partial}{\partial x_i} (N_a u_{a i}) = 0,$$  \[3.14\]

$$\frac{1}{N_a} \frac{\partial}{\partial t} (N_a u_{a a r}) + \tau_a u_{a a r} = -\sum_\delta \frac{N_\delta}{\sigma_{\delta a}} u_{\delta a r} + \frac{1}{N_a} \frac{\partial}{\partial x_i} (N_a P_{a i r})$$

$$- \left( \frac{e_a}{m_a} E_r + \frac{1}{m_a} F_{a r} \right) - \frac{e_a}{cm_a} \epsilon_{rjk} u_{a a j} H_k = 0,$$  \[3.15\]

$$\frac{1}{N_a} \frac{\partial}{\partial t} (N_a P_{a r r}) + \tau_a P_{a r r} = -\frac{2e_a}{cm_a} \epsilon_{rjk} P_{a r j} H_k$$

$$- \frac{1}{N_a} \frac{\partial}{\partial x_i} (N_a S_{a l r}) + 2 \left( \frac{e_a}{m_a} E_r + \frac{1}{m_a} F_{a r} \right) u_{a a r} + \sum_\delta \frac{N_\delta}{\sigma_{\delta a}} \left( u_{\delta a r}^2 + \frac{K T_{\delta a}}{m_a} \right)$$

$$= A_{a l r r},$$  \[3.16\]

$$\frac{1}{N_a} \frac{\partial}{\partial t} (N_a P_{a r s}) + \tau_a P_{a r s} = -\frac{e_a}{cm_a} \left( \epsilon_{rjk} P_{a r j} + \epsilon_{sjk} P_{a r j} \right) H_k$$

$$- \frac{1}{N_a} \frac{\partial}{\partial x_i} (N_a S_{a l r s}) + \frac{e_a}{m_a} \left( E_r u_{a a s} + E_s u_{a a r} \right) + \frac{1}{m_a} (F_{a r} u_{a a s} + F_{a s} u_{a a r})$$

$$+ \sum_\delta \frac{N_\delta}{\sigma_{\delta a}} u_{\delta a r} u_{\delta a s} \equiv B_{a r s},$$  \[3.17\]

$$\frac{1}{N_a} \frac{\partial}{\partial t} (N_a S_{a r r r}) + \tau_a S_{a r r r} = -\frac{3e_a}{cm_a} \epsilon_{rjk} S_{a r j} H_k$$

$$- \frac{1}{N_a} \frac{\partial}{\partial x_i} (N_a Q_{a r r r}) + 3 \left( \frac{e_a}{m_a} E_r + \frac{1}{m_a} F_{a r} \right) P_{a r r}$$

$$+ \sum_\delta \frac{N_\delta}{\sigma_{\delta a}} \left[ u_{\delta a r}^3 + \frac{3K T_{\delta a}}{m_a} u_{\delta a r} \right] \equiv A_{a r r r}.$$  \[3.18\]
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\[
\frac{1}{N_a} \frac{\partial}{\partial t} (N_a S_{arr}) + \frac{\partial}{\partial x_i} (N_a Q_{arr}) + \frac{e_a}{cm_a} (2 \epsilon_{ijk} S_{a_jrst} + \epsilon_{ijk} S_{ajr}) H_k
\]

\[
- \frac{1}{N_a} \frac{\partial}{\partial x_i} (N_a Q_{arr}) + 2 \left( \frac{e_a}{m_a} E_r + \frac{1}{m_a} F_{ar} \right) P_{ars} + \left( \frac{e_a}{m_a} E_s + \frac{1}{m_a} F_{as} \right) P_{ars}
\]

\[
+ \sum_{\delta} \frac{N_\delta}{\sigma_{\delta a}} \left[ \frac{1}{m_a} u_{\delta ar} u_{\delta as} + \frac{KT_{\delta a}}{m_a} u_{\delta as} \right] = B_{arrs}
\]

\[
\frac{1}{N_a} \frac{\partial}{\partial t} (N_a S_{arr}) + \frac{\partial}{\partial x_i} (N_a Q_{arr}) + \frac{e_a}{cm_a} (2 \epsilon_{ijk} S_{a_jrst} + \epsilon_{ijk} S_{ajr}) H_k
\]

\[
- \frac{1}{N_a} \frac{\partial}{\partial x_i} (N_a Q_{arr}) + \left( \frac{e_a}{m_a} E_r + \frac{1}{m_a} F_{ar} \right) P_{ars} + \left( \frac{e_a}{m_a} E_s + \frac{1}{m_a} F_{as} \right) P_{ars}
\]

\[
+ \sum_{\delta} \frac{N_\delta}{\sigma_{\delta a}} \left[ \frac{1}{m_a} u_{\delta ar} u_{\delta as} + \frac{KT_{\delta a}}{m_a} u_{\delta as} \right] = C_{arrs}
\]

\[
\frac{1}{N_a} \frac{\partial}{\partial t} (N_a Q_{arr}) + \frac{\partial}{\partial x_i} (N_a Q_{arr}) + \frac{4e_a}{cm_a} \epsilon_{ijk} Q_{ajr} H_k = A_{arrs}
\]

\[
\frac{1}{N_a} \frac{\partial}{\partial t} (N_a Q_{arr}) + \frac{\partial}{\partial x_i} (N_a Q_{arr}) + \frac{e_a}{cm_a} (\epsilon_{ijk} Q_{ajr} + 3 \epsilon_{ijk} Q_{ajr}) H_k = B_{arrs}
\]

\[
\frac{1}{N_a} \frac{\partial}{\partial t} (N_a Q_{arr}) + \frac{\partial}{\partial x_i} (N_a Q_{arr}) + \frac{e_a}{cm_a} (\epsilon_{ijk} Q_{ajr} + \epsilon_{ijk} Q_{ajr}) H_k = C_{arrs}
\]

\[
\frac{1}{N_a} \frac{\partial}{\partial t} (N_a Q_{arr}) + \frac{\partial}{\partial x_i} (N_a Q_{arr}) + \frac{2e_a}{cm_a} (\epsilon_{ijk} Q_{ajr} + \epsilon_{ijk} Q_{ajr}) H_k = D_{arrs}
\]

For convenience, we have recorded the lengthy expressions \( A_{arrs}, B_{arrs}, C_{arrs}, \) and \( D_{arrs} \) together with the solutions of \( Q_a \) in Appendix 4.

Equations [3.14] – [3.24] along with similar transport equations for components \( \beta \) and \( \gamma \) with \( e_\gamma = 0 \) govern the behaviour of the assembly.

4. **Transport Coefficients Deduced from Simple Flow Problems**

In order to understand the significance of the transport equations [3.14] – [3.24], we shall consider three simple flow problems.
In the case of a steady plane Couette flow with no heat flux, we replace the third order moments by the equivalent moments of lower order and obtain
\[ \frac{\partial S_{a21}}{\partial x_2} = -\tau_a P_{a12}, \quad [4.1] \]
or
\[ P_{a12} = -\left( \frac{2 KT_{aa}}{m_a \tau_a} \right) \left( \frac{1}{2} \frac{\partial u_{aa}}{\partial x_2} \right). \quad [4.2] \]

Hence, we conclude that in this simple problem the coefficient of viscosity is given by
\[ \mu = \frac{2 KT_{aa}}{m_a \tau_a}. \quad [4.3] \]

Similarly, if we consider steady one-dimensional heat flow in a fluid at rest, after replacing the fourth order moments by their equivalent lower order moments, we get
\[ \frac{\partial Q_{a111}}{\partial x_1} = -\tau_a S_{a111}, \quad [4.4] \]
or
\[ q_{a1} = \frac{1}{2} S_{a1} = -\left( \frac{5 K^2 T_{aa}}{m_a^2 \tau_a} \right) \frac{\partial T_{aa}}{\partial x_1}. \quad [4.5] \]

This leads to the coefficient of heat conductivity:
\[ k = \frac{5 K^2 T_{aa}}{m_a^2 \tau_a}. \quad [4.6] \]

Finally, considering Lorentz problem of steady, homogeneous flow of a macroscopically neutral mixture of charged particles in the presence of electric field \( E(E, 0, 0) \), the basic momentum equations reduce to
\[ \frac{N_{\beta} A_{\beta\alpha}}{\sigma_{\beta\alpha}} (u_{\beta\gamma} - u_{aa}) + \frac{N_{\gamma} A_{\gamma\alpha}}{\sigma_{\gamma\alpha}} (u_{\gamma\alpha} - u_{aa}) = -e_a E, \quad [4.7] \]
\[ \frac{N_{\gamma} A_{\gamma\beta}}{\sigma_{\gamma\beta}} (u_{\gamma\gamma} - u_{\beta\beta}) + \frac{N_{\alpha} A_{\alpha\beta}}{\sigma_{\alpha\beta}} (u_{aa} - u_{\beta\beta}) = -e_\beta E, \quad [4.8] \]
and
\[ \frac{N_{a} A_{\alpha\gamma}}{\sigma_{\alpha\gamma}} (u_{aa} - u_{\gamma\gamma}) + \frac{N_{\beta} A_{\beta\gamma}}{\sigma_{\beta\gamma}} (u_{\beta\gamma} - u_{\gamma\gamma}) = 0. \quad [4.9] \]
Eliminating \( u_{\gamma 1} \) and substituting for the current density

\[
J_1 = e_a N_a u_{a1} + e_\beta N_\beta u_{\beta 1}
\]  

[4.10]

and making use of the neutrality condition

\[
e_a N_a + e_\beta N_\beta = 0.
\]  

[4.11]

we obtain

\[
J_1 = \sigma E,
\]  

[4.12]

where

\[
\sigma = \frac{(e_\beta N_\beta - e_a N_a)^2}{N_a N_\beta} \times
\]

\[
\frac{N_a a_{a\gamma}}{\sigma_{a\gamma}} + N_\beta A_{\beta \gamma} / \sigma_{\beta \gamma}
\]  

[4.13]

Thus, we can interpret \( \sigma \) as the direct electrical conductivity of the macroscopically neutral medium. The expressions [4.3], [4.6] and [4.13] have the same structure as those given by Chapman and Cowling\(^1\) and Grad\(^13\).

5. STATIONARY NON-EQUILIBRIUM PHENOMENA

In this section we generalize the simple results obtained in the previous section to include all stationary phenomena such as density gradient, velocity gradients, stress variations, and heat flux vector. We shall as usual consider the system to be macroscopically neutral.

We shall first consider the expression for the current density \( J \). In order to find the contribution of the density and temperature gradients to the current, we shall replace the second order moments by their equivalent lower order moments. Then from the momentum equations [3.15], after straightforward calculation we get the expressions \( J_{11} \) for the current density parallel to the magnetic field \( H \) and \( J_{a\alpha} \) and \( J_{\beta\alpha} \) for perpendicular component of the current densities due to electrons and ions:

\[
J_{11} = \sigma E_{11} + \sigma_\alpha \left[ \nabla_{11}(K T_{a\alpha}) + KT_{a\alpha} \nabla_{11} \log N_a - F_{a11} \right]
\]

\[
+ \sigma_\beta \left[ \nabla_{11}(K T_{\beta\beta}) + KT_{\beta\beta} \nabla_{11} \log N_\beta - F_{\beta11} \right],
\]  

[5.1]

\[
J_{a\alpha} = \frac{c(e_a N_a - e_\beta N_\beta)}{2 H^2} E \times H
\]
\[- \frac{c N_a}{H^2} \left( \nabla (KT_{aa}) + KT_{aa} \nabla \log N_a - F_a \right) \times H \]

\[- \frac{c N_\gamma}{D_2 H^2} \cdot \frac{N_a A_{a\gamma}}{\sigma_{\beta \gamma}} \left[ \nabla (KT_{\gamma\gamma}) + KT_{\gamma\gamma} \nabla \log N_\gamma - F_\gamma \right] \times H \]

\[+ \frac{2 c^2 N_\alpha N_\beta D_1}{(e_\beta N_\beta - e_a N_a) D_2 H^2} \left[ \nabla \left( N_\alpha KT_{aa} + N_\beta KT_{\beta\beta} + N_\gamma KT_{\gamma\gamma} \right) \right. \]

\[\left. - \left( N_\alpha F_a + N_\beta F_\beta + N_\gamma F_\gamma \right) \right] \]

\[\frac{c (e_\beta N_\beta - e_a N_a)}{2 H^2} E \times H \]

\[- \frac{c N_\beta}{H^2} \left( \nabla (KT_{\beta\beta}) + KT_{\beta\beta} \nabla \log N_\beta - F_\beta \right) \times H \]

\[- \frac{c N_\gamma}{D_2 H^2} \cdot \frac{N_\beta A_{\beta\gamma}}{\sigma_{\beta \gamma}} \left[ \nabla (KT_{\gamma\gamma}) + KT_{\gamma\gamma} \nabla \log N_\gamma - F_\gamma \right] \times H \]

\[+ \frac{2 c^2 N_\alpha N_\beta D_1}{(e_a N_a - e_\beta N_\beta) D_2 H^2} \left[ \nabla \left( N_\alpha KT_{aa} + N_\beta KT_{\beta\beta} + N_\gamma KT_{\gamma\gamma} \right) \right. \]

\[\left. - \left( N_\alpha F_a + N_\beta F_\beta + N_\gamma F_\gamma \right) \right] \]

where

\[\sigma = \frac{(e_\beta N_\beta - e_a N_a)^2}{4 N_\alpha N_\beta} \cdot \frac{D_2}{D_1}, \quad [5.4] \]

\[\sigma_a = \frac{(e_\beta N_\beta - e_a N_a)}{2 D_1} \cdot \frac{A_{a\gamma}}{\sigma_{a\gamma}}, \quad [5.5] \]

\[\sigma_\beta = \frac{(e_a N_a - e_\beta N_\beta)}{2 D_1} \cdot \frac{A_{\beta\gamma}}{\sigma_{\beta\gamma}}, \quad [5.6] \]
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\[ D_1 = \frac{N_a A_{\beta\alpha} A_{\gamma\gamma}}{\sigma_{\beta\alpha} \sigma_{\gamma\gamma}} + \frac{N_b A_{\gamma\gamma} A_{\beta\alpha}}{\sigma_{\gamma\beta} \sigma_{\beta\alpha}} + \frac{N_c A_{\alpha\gamma} A_{\gamma\beta}}{\sigma_{\alpha\gamma} \sigma_{\gamma\beta}}, \quad [5.7] \]

and

\[ D_2 = \frac{N_a A_{\alpha\gamma}}{\sigma_{\alpha\gamma}} + \frac{N_b A_{\beta\gamma}}{\sigma_{\beta\gamma}}. \quad [5.8] \]

These expressions clearly exhibit the effect of temperature and density gradients as well as that of other external forces on current density. From [5.1] we conclude that \( \sigma \) is the direct electrical conductivity along the magnetic field, while \( \sigma_a \) and \( \sigma_b \) can termed as diffusion coefficients. We note that the expression for \( \sigma \) is the same as [4.13]. However, the dependence of current density in [5.2] and [5.3] are more complicated.

In order to deal with other physical variables we shall choose the coordinate axes in such a way that \( \mathbf{H} = (0, 0, H) \) without loss of generality.

Solving the stress equations [3.16] and [3.17] and denoting the electron and ion gyrofrequencies by \( \omega_a \) and \( \omega_\beta \), given by

\[ \omega_a = \frac{e_a H}{c m_a}, \quad \omega_\beta = \frac{e_\beta H}{c m_\beta}, \quad [5.9] \]

we have

\[ P_{a33} = \frac{1}{\tau_a}, \quad [5.10] \]

\[ P_{a11} = \frac{1}{\Delta_2 \tau_a} \left[ 2 \omega_a \tau_a B_{a12} + (\tau_a^2 + 2 \omega_a^2) A_{a11} + 2 \omega_a^2 A_{a22} \right] \quad [5.11] \]

\[ P_{a22} = \frac{1}{\Delta_2 \tau_a} \left[ -2 \omega_a \tau_a B_{a12} + 2 \omega_a^2 A_{a11} + (\tau_a^2 + 2 \omega_a^2) A_{a22} \right], \quad [5.12] \]

\[ P_{a12} = \left( \frac{1}{\Delta_2} \right) \left[ \tau_a B_{12} - \omega_a (A_{a11} - A_{a22}) \right], \quad [5.13] \]

\[ P_{a23} = \left( \frac{1}{\Delta_1} \right) \left[ \tau_a B_{a23} - \omega_a B_{a31} \right], \quad [5.14] \]

\[ P_{a31} = \left( \frac{1}{\Delta_1} \right) \left[ \omega_a B_{a23} + \tau_a B_{a31} \right], \quad [5.15] \]

where

\[ \Delta_1 = \tau_a^2 + \omega_a^2, \quad \Delta_2 = \tau_a^2 + 4 \omega_a^2. \quad [5.16] \]

We note that \( P_{a33} \) is not affected by the magnetic field and

\[ P_{a11} + P_{a22} + P_{a33} = \left[ \frac{1}{\tau_a} \right] (A_{a11} + A_{a22} + A_{a33}) \]

is also independent of the magnetic field. Thus, the stress component along the magnetic field acting on a plane perpendicular to the magnetic field and the isotropic pressure and hence the total internal energy are unaffected by the magnetic field.
Concentrating on the dependence of the stresses on the gradients of velocity components, temperature and the density, we find that

$$P_{a33} = -\mu^{(0)}_{a33} e_{a33} - \mu^{(1)}_{a33} \nabla T_{aa} - \mu^{(2)}_{a33} \nabla_3 N_a.$$  \[5.17\]

where

$$\mu^{(0)}_{a33} = \frac{2 K T_{aa}}{m_a \tau_a},$$  \[5.18\]

$$\mu^{(2)}_{a33} = \frac{K}{m_a \tau_a} \begin{bmatrix} u_{aa1} & u_{aa2} & u_{aa3} \end{bmatrix},$$  \[5.19\]

$$\mu^{(0)}_{a33} = \frac{u_{aa3}}{N_a} \mu^{(0)}_{a33}.$$  \[5.20\]

We can call these $\mu$ matrices the generalized viscosity matrix. $\mu^{(0)}_{a33}$ has the same form as [4.3] and is unaffected by the magnetic field.

Similarly, we have

$$\begin{pmatrix} P_{a33} \\ P_{a31} \end{pmatrix} = -\mu^{(0)}_{a3} \begin{pmatrix} e_{a33} \\ e_{a31} \end{pmatrix} - \mu^{(1)}_{a3} \nabla T_{aa} - \mu^{(2)}_{a3} \nabla N_a.$$  \[5.21\]

where

$$\mu^{(0)}_{a3} = \frac{2 K T_{aa}}{m_a \Delta_1} \begin{pmatrix} \tau_a & -\omega_a \\ \omega_a & \tau_a \end{pmatrix},$$  \[5.22\]

$$\mu^{(1)}_{a3} = \frac{K}{m_a \Delta_1} \begin{pmatrix} -\omega_a u_{aa3} & \tau_a u_{aa3} & \tau_a u_{aa1} - \omega_a u_{aa1} \\ \tau_a u_{aa3} & \omega_a u_{aa3} & \omega_a u_{aa2} + \tau_a u_{aa1} \end{pmatrix},$$  \[5.23\]

$$\mu^{(2)}_{a3} = (T_{aa}/N_a) \mu^{(1)}_{a3}.$$  \[5.24\]

Regarding the viscosity matrices associated with the stresses along the magnetic field acting on planes containing the magnetic field, we note that asymmetry is caused by the magnetic field. Moreover, the primary viscosity coefficient is reduced by the magnetic field since the diagonal terms of $\mu^{(0)}_{a3}$ can be written as

$$\frac{2 K T_{aa}}{m_a \tau_a} \left( 1 - \frac{\omega_a^2}{\tau_a^2 + \omega_a^2} \right).$$  \[5.25\]

The stresses due to temperature gradient and density gradient are proportional to each other.
Finally, writing
\[
\begin{bmatrix}
P_{a11} \\
P_{a12} \\
P_{a22}
\end{bmatrix} = -\mu_a^{(0)} \begin{bmatrix}
e_{a11} \\
e_{a12} \\
e_{a22}
\end{bmatrix} - \mu_a^{(1)} \nabla T_{a2} - \mu_a^{(2)} \nabla N_{a2}
\]
we have
\[
\mu_a^{(0)} = \frac{2 K T_{a2}}{m_a \tau_a \Delta_2} \begin{bmatrix}
\tau_a + 2 \omega_a^2 & 2 \omega_a \tau_a & 2 \omega_a^2 \\
-\omega_a \tau_a & \tau_a^2 & -\omega_a \tau_a \\
2 \omega_a^2 & -2 \omega_a \tau_a & \tau_a^2 + 2 \omega_a^2
\end{bmatrix},
\]
\[
\mu_a^{(1)} = \frac{K}{m_a \gamma_a \Delta_2} \begin{bmatrix}
2 \omega_a \tau_a u_{aa2} + (3 \tau_a^2 + 8 \omega_a^2) u_{aa1} & -2 \omega_a \tau_a u_{aa2} + (\tau_a^2 + 8 \omega_a^2) u_{aa1} & \Delta_2 u_{aa3} \\
\tau_a u_{aa2} - 2 \omega_a \tau_a u_{aa1} & 2 \omega_a \tau_a u_{aa2} + \tau_a^2 u_{aa1} & 0 \\
+2 \omega_a \tau_a u_{aa1} + (\tau_a^2 + 8 \omega_a^2) u_{aa2} & -2 \omega_a \tau_a u_{aa1} + (3 \tau_a^2 + 8 \omega_a^2) u_{aa2} & \Delta_2 u_{aa3}
\end{bmatrix}
\]
\[
\mu_a^{(2)} = \frac{T_{a2}}{N} \mu_a^{(1)} ,
\]

Thus, considering the viscosity matrix corresponding to the stresses perpendicular to the magnetic field, we conclude that the magnetic field introduces anisotropy. Considering the diagonal terms we see that for \( P_{a11} \) and \( P_{a22} \) the corresponding viscosity coefficient is
\[
\frac{2 K T_{a2}}{m_a \tau_a} \left( 1 - \frac{2 \omega_a^2}{\tau_a^2 + 4 \omega_a^2} \right) ,
\]
while corresponding to \( P_{a12} \) we have
\[
\frac{2 K T_{a2}}{m_a \tau_a} \left( 1 - \frac{4 \omega_a^2}{\tau_a^2 + 4 \omega_a^2} \right) .
\]
Once again the stresses due to density gradient are proportional to stresses due to temperature gradient.

Comparing the expressions [5.18], [5.25], [5.30] and [5.31], we conclude that the magnetic field introduces intense anisotropy. Further, the coefficients of viscosity in the plane perpendicular to the magnetic field are less than the the viscosity in a plane containing the magnetic field.

From the other terms we can deduce the effect of electric field, external forces, collisional transfers, etc., on the stresses.
Solving the equations [3.18] – [3.20] for heat flux tensor, we have

\[ S_{a333} = \frac{1}{\tau_a} A_{a333}, \]  

\[ S_{a331} = \frac{1}{\Delta_1} \left[ \tau_a B_{a331} + \omega_a B_{a332} \right], \]  

\[ S_{a332} = \frac{1}{\Delta_1} \left[ \tau_a B_{a331} - \omega_a B_{a332} \right], \]  

\[ S_{a123} = \frac{1}{\Delta_2} \left[ \tau_a C_{a123} + \omega_a (B_{a223} - B_{a113}) \right], \]  

\[ S_{a113} = \frac{1}{\tau_a \Delta_2} \left[ 2 \omega_a \tau_a C_{a123} + (\tau_a^2 + 2 \omega_a^2) B_{a113} + 2 \omega_a^2 B_{a223} \right], \]  

\[ S_{a223} = \frac{1}{\tau_a \Delta_2} \left[ -2 \omega_a \tau_a C_{a123} + 2 \omega_a^2 B_{a113} + (\tau_a^2 + 2 \omega_a^2) B_{a223} \right], \]  

\[ S_{a112} = \frac{1}{\Delta_1 \Delta_3} \left[ (\tau_a^2 + 3 \omega_a^2) (\tau_a B_{a112} - \omega_a A_{a111}) + 2 \omega_a \tau_a (\tau_a B_{a221} + \omega_a A_{a222}) \right], \]  

\[ S_{a221} = \frac{1}{\Delta_1 \Delta_3} \left[ 2 \omega_a \tau_a (\tau_a A_{a111} - \omega_a B_{a112}) + (\tau_a^2 + 3 \omega_a^2) (\tau_a B_{a221} + \omega_a A_{a222}) \right], \]  

\[ S_{a111} = \frac{1}{\Delta_1 \Delta_3} \left[ 3 \omega_a (\tau_a^2 + 3 \omega_a^2) B_{a112} + \tau_a (\tau_a^2 + 7 \omega_a^2) A_{a111} + 6 \omega_a^2 (\tau_a B_{a221} + \omega_a A_{a222}) \right], \]  

\[ S_{a222} = \frac{1}{\Delta_1 \Delta_3} \left[ 6 \omega_a^2 (\tau_a B_{a112} - \omega_a A_{a111}) - 3 \omega_a (\tau_a^2 + 3 \omega_a^2) B_{a221} + \tau_a (\tau_a^2 + 7 \omega_a^2) A_{a222} \right], \]  

where \[ \Delta_3 = \tau_a^2 + 9 \omega_a^2. \]

Concentrating mainly on the temperature and density gradients of the heat flux vector

\[ S_{ar} = \frac{1}{2} S_{arll}, \]

we find that we can write

\[ S_{a3} = - \frac{5 K^2 T_{aa}}{m_a^2 \tau_a} \frac{\partial T_{aa}}{\partial x_3} - \frac{5 K^2 T_{aa}^2}{2 m_a^2 N_a \tau_a} \frac{\partial N_a}{\partial x_3}, \]

and

\[ \begin{pmatrix} S_{a1} \\ S_{a2} \end{pmatrix} = - K_a^{(0)} \begin{pmatrix} (\partial T_{aa}/\partial x_1) \\ (\partial T_{aa}/\partial x_2) \end{pmatrix} - K_a^{(1)} \begin{pmatrix} (\partial N_a/\partial x_1) \\ (\partial N_a/\partial x_2) \end{pmatrix}, \]

where

\[ K_a^{(0)} = \begin{pmatrix} 5 K^2 T_{aa} \end{pmatrix} \begin{pmatrix} \tau_a \\ - \omega_a \end{pmatrix}, \]

and

\[ K_a^{(1)} = \frac{T_{aa}}{2 N_a} K_a^{(0)}. \]
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From [5.44] we conclude that the heat conductivity $k_{a\alpha}^{(0)}$ coefficient in the direction of magnetic field is unaltered and is the same as the expression [4.5]. But the heat conductivity tensor transverse to magnetic field is modified due to the presence of the magnetic field. The collisional contribution to heat conductivity is given by

$$\frac{5 k^2 T_{a\alpha}}{m_\alpha^2 \tau_\alpha} \left( 1 - \frac{\omega_\alpha^2}{\omega_\alpha^2 + \tau_\alpha^2} \right),$$

which decreases as the magnetic field increases. The heat conductivity tensor $k_{a\alpha}^{(1)}$ arising out of the density gradient is directly proportional to the heat conductivity tensor $k_{a\alpha}^{(0)}$ due to temperature gradient both along and perpendicular to the magnetic field as seen from [5.44] and [4.47].

We emphasize here that the generalized stress and thermal transport coefficients have in their denominators factors of the type $\tau_\alpha^2 + \omega_\alpha^2$, $\tau_\alpha^2 + 4 \omega_\alpha^2$, $\tau_\alpha^2 + 9 \omega_\alpha^2$. Proceeding in a similar manner we have established the corresponding coefficients for the fourth order moments (Appendix IV). The only remark of interest about the fourth order moments is that the denominators have $\tau_\alpha^2 + 16 \omega_\alpha^2$ as an additional factor Generalizing we can state as follows: For any $n$-th order moment, if all the suffixes are along the magnetic field direction then it contains $(1/\tau_\alpha)$ only; if $(n - 1)$ suffixes are in the direction of magnetic field it has a factor $(1/\Delta_1)$ and generally if $(n - r)$ suffixes are in the direction of the magnetic field, it has a factor $(1/\Delta_r)$ where

$$\Delta_r = \tau_\alpha^2 + r^2 \omega_\alpha^2.$$  

This point is of great importance while establishing the relaxation times in §6, as it leads to an explanation of Gross gaps [18].

6. Relaxation Problem

Finally, we shall consider the relaxation problem. Following Bhatnagar [19] we shall suppose that the physical quantities depend only on time and there is no external force field excepting the magnetic field. Further, for simplicity, we shall suppose that the axes are so chosen that $H = (0, 0, H)$. From the zeroth order continuity equations, we conclude that

$$N_\alpha = \text{constant}.$$  

The relaxation times $p$ for the velocity components are governed by the equations

$$p \left( \frac{N_\beta A_{\beta\alpha} m_\alpha}{\sigma_{\beta\alpha}} + \frac{N_\gamma A_{\gamma\alpha} m_\alpha}{\sigma_{\gamma\alpha}} \right) u_{a\alpha \alpha r} - \omega_\alpha \epsilon_{rij} u_{a\alpha j}$$

$$= \frac{N_\beta A_{\beta\alpha} m_\alpha}{\sigma_{\beta\alpha}} u_{\beta\beta r} + \frac{N_\gamma A_{\gamma\alpha} m_\alpha}{\sigma_{\gamma\alpha}} u_{\gamma\gamma r},$$  

[6.2]
Neglecting the square of the collisional terms, for the components along the magnetic field, we have

\[
p = - \frac{1}{2} \left[ \frac{N_\alpha A_{\alpha \beta} m_\beta}{\sigma_{\alpha \beta}} + \frac{N_\alpha A_{\alpha \gamma} m_\gamma}{\sigma_{\alpha \gamma}} + \frac{N_\beta A_{\beta \gamma} m_\gamma}{\sigma_{\beta \gamma}} + \frac{N_\beta A_{\beta \alpha} m_\alpha}{\sigma_{\beta \alpha}} + \frac{N_\gamma A_{\gamma \alpha} m_\alpha}{\sigma_{\gamma \alpha}} \right. \\
\left. + \frac{N_\gamma A_{\gamma \beta} m_\beta}{\sigma_{\gamma \beta}} \right] \quad [6.3]
\]

which is unaffected by the magnetic field.

For the transverse components, we have three modes given by

\[
p_1 = - \left( \frac{N_\alpha A_{\alpha \gamma} m_\alpha}{\sigma_{\alpha \gamma}} + \frac{N_\beta A_{\beta \gamma} m_\gamma}{\sigma_{\beta \gamma}} \right),
\]

\[
p_2 = \pm i \omega_\beta - \left( \frac{N_\gamma A_{\gamma \beta} m_\beta}{\sigma_{\gamma \beta}} + \frac{N_\alpha A_{\alpha \gamma} m_\gamma}{\sigma_{\alpha \gamma}} \right),
\]

\[
p_3 = \pm i \omega_\alpha - \left( \frac{N_\beta A_{\beta \alpha} m_\alpha}{\sigma_{\beta \alpha}} + \frac{N_\gamma A_{\gamma \alpha} m_\alpha}{\sigma_{\gamma \alpha}} \right). \quad [6.4]
\]

We note here that the self-collisions do not contribute to these relaxation times.

Proceeding in the similar fashion, the relaxation time for the temperature is given by

\[
p = - \frac{1}{2} \left( \frac{N_\alpha B_{\alpha \beta}}{\sigma_{\alpha \beta}} + \frac{N_\alpha B_{\alpha \gamma}}{\sigma_{\alpha \gamma}} + \frac{N_\beta B_{\beta \alpha}}{\sigma_{\beta \alpha}} + \frac{N_\beta B_{\beta \gamma}}{\sigma_{\beta \gamma}} + \frac{N_\gamma B_{\gamma \alpha}}{\sigma_{\gamma \alpha}} + \frac{N_\gamma B_{\gamma \beta}}{\sigma_{\gamma \beta}} \right), \quad [6.5]
\]

while the off-diagonal terms of the pressure tensor have the relaxation times

\[
p_1 = - \tau_\alpha, \quad p_2 = \pm i \omega_\alpha - \tau_\alpha, \quad p_3 = \pm 2 i \omega_\alpha \quad - \tau_\alpha. \quad [6.6]
\]

The relaxation times of the third order moments are determined by equations identical in form to [5.32] - [5.41] with \( \tau_\alpha \) replaced by \( \tau_\alpha + p \). Hence proceeding as in the calculation of transport coefficients we obtain the following relaxation times.

\[
p = - \tau_\alpha \quad \text{for } S_{a333},
\]

\[
p = \pm i \omega_\alpha - \tau_\alpha \quad \text{for } S_{a331} \text{ and } S_{a332},
\]

\[
\]
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\[ \rho_1 = -\tau_\alpha, \rho_2 = \pm 2i\omega_\alpha - \tau_\alpha \text{ for } S_{\alpha 123}, S_{\alpha 113}, \text{ and } S_{\alpha 223}, \]

\[ \rho_1 = \pm i\omega_\alpha - \tau_\alpha, \rho_2 = \pm 3i\omega_\alpha - \tau_\alpha \]

for \( S_{\alpha 111}, S_{\alpha 112}, S_{\alpha 122}, \text{ and } S_{\alpha 222}. \) [6.7]

Finally, we shall record the relaxation times for the fourth order moments.

\[ \rho = -\tau_\alpha \text{ for } Q_{\alpha 3333}, \]

\[ \rho = \pm i\omega_\alpha - \tau_\alpha \] for \( Q_{\alpha 3331} \) and \( Q_{\alpha 3332}, \)

\[ \rho_1 = -\tau_\alpha, \rho_2 = \pm 2i\omega_\alpha - \tau_\alpha \text{ for } Q_{\alpha 3311}, Q_{\alpha 3312}, \text{ and } Q_{\alpha 3322}, \]

\[ \rho_1 = \pm i\omega_\alpha - \tau_\alpha, \rho_2 = \pm 3i\omega_\alpha - \tau_\alpha \]

for \( Q_{\alpha 3111}, Q_{\alpha 3112}, Q_{\alpha 3122}, \text{ and } Q_{\alpha 3222}. \) [6.8]

From the above expressions, the following plausible explanation of Gross gaps can be given. If we take the dependence of the physical quantities as \( e^{i\omega t} \) instead of \( e^{\pm t} \) then the amplitudes of the moments of the distribution function will be obtained by putting \( \tau_\alpha + i\omega \) instead of \( \tau_\alpha \) in § 5, so that the complex frequency of oscillation will be determined by expressions of the type \( p = \pm n\omega_\alpha + i\omega_\alpha \). Correspondingly, the amplitudes of \( n \)th order moments having no suffix along the magnetic field will contain in their denominators an expression of the type \( (p - i\omega_\alpha)^2 - n^2\omega_\alpha^2 \). Thus, in the absence of collisions \( (\tau_\alpha = 0) \) these amplitudes will become infinity when \( p = n\omega_\alpha \) or \( n\omega_\beta \). It appears, therefore, that the wave is dissipated away on account of making \( n \)th and higher order moments infinity. This result was obtained by Gross for one component assembly and extended to multicomponent assembly by Bhatnagar and Devanathan on the basis of kinetic equation. Here we have obtained the physical mechanism responsible for the decay of such waves in detail.

APPENDIX I.

Average number of Collision

The average number of collisions between \( \beta \) and \( \alpha \) components is defined by

\[ N_\beta N_\alpha / \sigma_{\beta \alpha} = \int \int f_\beta f_\alpha g_{\beta \alpha} b db dt d\xi_\beta d\xi_\alpha \] [A1.1]

The exact expressions can be evaluated by using the expansion for \( f_\alpha \) and \( f_\beta \). But as can be easily shown from the kinetic equations [2.9] that for small deviations from equilibrium position only the equilibrium value of \( N_\beta N_\alpha / \sigma_{\beta \alpha} \) is necessary. Correspondingly we choose the equilibrium Maxwellian distributions

\[ f_\alpha = N_\alpha \left( \frac{m_\alpha}{2\pi K T_{\alpha \alpha}} \right)^{3/2} \exp \left[ -\frac{m_\alpha}{2 K T_{\alpha \alpha}} \xi_\alpha^2 \right], \] [A1.2]
with similar expression for \( f_\beta \). Then [AI.1] reduces to

\[
\frac{1}{\sigma_{\beta a}} = \left( \frac{m_a m_\beta}{K^2 T_{aa} T_{\beta \beta}} \right)^{3/2} \int \int \exp \left[ -\frac{m_a}{2 K T_{aa}} \xi_a^2 - \frac{m_\beta}{2 K T_{\beta \beta}} \xi_\beta^2 \right] \times \]

\[
g_{\beta a} \ d db \ d de \ d \xi_a \ d \xi_\beta \tag{AI.3}
\]

The integration over the impact parameter \( b \) can be carried out exactly on the same lines as Chapman and Cowling\(^1\), for the force law \( |F_{\beta a}| = K_{\beta a} / r^2 \). In order to ensure convergence we have to introduce cut-off. In terms of the non-dimensional impact parameter \( v_0 \) we have

\[
\frac{1}{\sigma_{\beta a}} = \frac{1}{2 K \left( \frac{m_a m_\beta}{T_{aa} T_{\beta \beta}} \right)^{3/2}} \left[ \frac{(m_a m_\beta) K_{\beta a}}{m_a m_\beta} \right]^{\frac{2}{s-1}} (v_{ou}^2 - v_{oi}^2) \times \]

\[
\int \int \exp \left[ -\frac{m_a}{2 K T_{aa}} \xi_a^2 - \frac{m_\beta}{2 K T_{\beta \beta}} \xi_\beta^2 \right] g_{\beta a} \left( \frac{x}{s-1} \right) d \xi_a d \xi_\beta \tag{AI.4}
\]

By changing the variables to

\[
g_{\beta a} = \vec{\xi}_a - \vec{\xi}_\beta \quad \text{and} \quad \mathbf{v}_{\beta a} = \vec{\xi}_\beta + \vec{\xi}_a
\]

and carrying out the elementary integrations we get finally

\[
\frac{1}{\sigma_{\beta a}} = \frac{1}{(2\pi)^{2}} \left[ \frac{T_{aa}}{m_a} + \frac{T_{\beta \beta}}{m_\beta} \right]^{\frac{2s-4}{s-1}} \left[ \frac{(m_a + m_\beta) K_{\beta a}}{m_a m_\beta} \right]^{\frac{2}{s-1}} (v_{ou}^2 - v_{oi}^2) \times F(s) \tag{AI.5}
\]

where

\[
F(s) = \Gamma \left( \frac{2s-4}{s-1} \right) \quad \text{if} \ s \neq 2 \quad \text{and} \quad F(s) = \int_{x^*}^{\infty} e^{-x} dx \quad \text{if} \ s = 2,
\]

\( x^* \) being suitable lower cut-off.

**APPENDIX II**

The Hermite Polynomials of first four degrees and the first four moments

\[
H^{(0)}_a = 1, \quad H^{(1)}_a = v_{ai}, \quad H^{(2)}_{aij} = v_{aj} v_{ai} - \delta_{ij}, \\
H^{(3)}_{aijk} = v_{al} v_{aj} v_{ak} - (v_{ai} \delta_{jk} + v_{aj} \delta_{ik} + v_{ak} \delta_{ij}), \\
H^{(4)}_{aijkl} = v_{ai} v_{aj} v_{ak} v_{al} - (v_{ai} v_{aj} \delta_{kl} + v_{al} v_{aj} \delta_{ik} + v_{ak} v_{aj} \delta_{il} + v_{al} v_{ak} \delta_{ij} + v_{ai} v_{al} \delta_{ik} + v_{ai} v_{ak} \delta_{il} + v_{al} v_{aj} \delta_{ik} + v_{aj} v_{ak} \delta_{il} + \delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}).
\]
Transport processes in a multi-component assembly

\[ \alpha_{a}^{(0)} = 1, \]
\[ \alpha_{a}^{(1)} = \left( \frac{m_a}{K T_{aa}} \right)^{1} u_{aai}, \]
\[ \alpha_{aii}^{(2)} = \frac{1}{2} \left[ \left( \frac{m_a}{K T_{aa}} \right) P_{aii} - 1 \right], \]
\[ \alpha_{aij}^{(2)} = \left( \frac{m_a}{K T_{aa}} \right) P_{aij}, \]
\[ \alpha_{aii}^{(3)} = \frac{1}{6} \left[ \left( \frac{m_a}{K T_{aa}} \right)^{3/2} S_{alii} - 3 \left( \frac{m_a}{K T_{aa}} \right)^{1/2} u_{aai} \right], \]
\[ \alpha_{aij}^{(3)} = \frac{1}{2} \left[ \left( \frac{m_a}{K T_{aa}} \right)^{3/2} S_{alij} - \left( \frac{m_a}{K T_{aa}} \right)^{1/2} u_{aaj} \right], \]
\[ \alpha_{aij}^{(3)} = \left( \frac{m_a}{K T_{aa}} \right)^{3/2} S_{aljk}, \]
\[ \alpha_{aiii}^{(4)} = \frac{1}{2 \pi} \left[ \left( \frac{m_a}{K T_{aa}} \right)^{2} Q_{aaiii} - 6 \left( \frac{m_a}{K T_{aa}} \right) P_{aaii} + 3 \right], \]
\[ \alpha_{aij}^{(4)} = \frac{1}{6} \left[ \left( \frac{m_a}{K T_{aa}} \right)^{2} Q_{aaij} - \left( \frac{m_a}{K T_{aa}} \right) P_{aij} \right], \]
\[ \alpha_{aij}^{(4)} = \frac{1}{2} \left[ \left( \frac{m_a}{K T_{aa}} \right)^{2} Q_{aaijk} - \left( \frac{m_a}{K T_{aa}} \right) P_{aik} \right], \]
\[ \alpha_{aijj}^{(4)} = \frac{1}{4} \left[ \left( \frac{m_a}{K T_{aa}} \right)^{2} Q_{aijj} - \left( \frac{m_a}{K T_{aa}} \right) (P_{aii} + P_{ajj}) + 1 \right]. \]

Appendix 3.

Number of Distinct Components of \( a^{(n)} \)

Since we are dealing with generalized Hermite polynomials of the velocity in three dimensional Euclidean space, the number of distinct components of \( a^{(n)} \) is nothing but the number of partitions of \( n \) each partition containing terms not exceeding three in number. Denoting this by \( P_3(n) \), we have from elementary number theory.\(^{20}\)
\[ P_3(n) = \frac{1}{n!} \left\{ \frac{d^n}{dx^n} \left[ \frac{1}{(1-x)(1-x^2)(1-x^3)} \right] \right\}_{x=0} = \frac{1}{72} \left[ 47 + 36n + 6n^2 + \left( -1 \right)^n \left( 9 + 16\cos\frac{n\pi}{3} \right) \right], \]

by actual differentiation.

**APPENDIX 3**

*Solution of fourth order moments*

\[ \tau_a Q_{a3333} = A_{a3333}, \]

\[ \Delta_1 Q_{a3331} = \tau_a B_{a3331} + \omega_a B_{a3332}, \]

\[ \Delta_1 Q_{a3332} = \tau_a B_{a3332} - \omega_a B_{a3331}. \]

\[ \tau_a \Delta_2 Q_{a3311} = (\tau_a^2 + 2\omega_a^2) D_{a3311} + 2\omega_a (\tau_a C_{a3312} + \omega_a D_{a2233}), \]

\[ \Delta_2 Q_{a3312} = -\omega_a D_{a3311} + \tau_a C_{a3312} + \omega_a D_{a2233}, \]

\[ \tau_a \Delta_2 Q_{a3322} = 2\omega_a (\omega_a D_{a3311} - \tau_a C_{a3312}) + (\tau_a^2 + 2\omega_a^2) D_{a2233}, \]

\[ \Delta_1 \Delta_3 Q_{a2223} = \tau_a (\tau_a^2 + 7\omega_a^2) B_{a2223} - 3\omega_a (\tau_a^2 + 3\omega_a^2) C_{a2231} + 6\omega_a^2 (\tau_a C_{a1123} - \omega_a B_{a1113}), \]

\[ \Delta_1 \Delta_3 Q_{a2213} = (\tau_a^2 + 3\omega_a^2) (\omega_a B_{a2223} + \tau_a C_{a2231}) - 2\omega_a \tau_a (\tau_a C_{a1123} - \omega_a B_{a1113}), \]

\[ \Delta_1 \Delta_3 Q_{a2113} = 2\omega_a \tau_a (\omega_a B_{a2223} + \tau_a C_{a2231}) + (\tau_a^2 + 3\omega_a^2) (\tau_a C_{a1123} - \omega_a B_{a1113}), \]

\[ \Delta_1 \Delta_3 Q_{a1113} = 6\omega_a^2 (\omega_a B_{a2223} + \tau_a C_{a2231}) + 3\omega_a (\tau_a^2 + 3\omega_a^2) C_{a1123} + \tau_a (\tau_a^2 + 7\omega_a^2) B_{a1113}, \]

\[ \tau_a \Delta_2 \Delta_4 Q_{a1111} = (\tau_a^4 + 16\tau_a^2\omega_a^2 + 24\omega_a^4) A_{a1111} + 4\omega_a \tau_a (\tau_a^2 + 10\omega_a^2) B_{a1112} + 12\omega_a^3 (\tau_a^2 + 4\omega_a^2) D_{a1122} + 24\omega_a^4 (\tau_a B_{a2221} + \omega_a A_{a2222}), \]

\[ \tau_a \Delta_2 \Delta_4 Q_{a1112} = \tau_a (\tau_a^2 + 10\omega_a^2) (\tau_a B_{a1112} - \omega_a A_{a1111}) + 7\omega_a \tau_a (\tau_a^2 + 4\omega_a^2) D_{a1122} + 6\omega_a^2 \tau_a (\tau_a B_{a2221} + \omega_a A_{a2222}), \]

\[ \tau_a \Delta_2 \Delta_4 Q_{a1122} = 2\omega_a (\tau_a^2 + \omega_a^2) (\omega_a A_{a1111} - \tau_a B_{a1112}) + (\tau_a^2 + 4\omega_a^2)^2 D_{a1122} + 2\omega_a (\tau_a^2 + 4\omega_a^2) (\tau_a B_{a2221} + \omega_a A_{a2222}), \]
Transport processes in a multi-component assembly

\[ \Delta_2 \Delta_4 Q_{a2221} = 6 \omega_a^2 (\tau_a B_{a1112} - \omega_a A_{a1111}) \\
- 3 \omega_a (\tau_a^2 + 4 \omega_a^2) D_{a1122} + \tau_a (\tau_a^2 + 10 \omega_a^2) (\tau_a B_{a2221} + \omega_a A_{a2222}), \]

\[ \tau_a \Delta_2 \Delta_4 Q_{a2222} = 24 \omega_a^3 (\omega_a A_{a1111} - \tau_a B_{a1112}) \\
+ 12 \omega_a^2 (\tau_a^2 + 4 \omega_a^2) D_{a1122} - 4 \omega_a \tau_a (\tau_a^2 + 10 \omega_a^2) B_{a2221} \\
+ (\tau_a^4 + 16 \omega_a^2 \tau_a^2 + 24 \omega_a^4) A_{a2222}, \]

where

\[ A_{arr} = - \left( \frac{K T_{aa}}{m_a} \right) \left[ 4 \frac{\partial S_{arr}}{\partial x_r} + 4 S_{arr} \frac{\partial}{\partial x_r} \left( \log \frac{K T_{aa}}{m_a} \right) \right] \\
+ 6 S_{arr} \frac{\partial}{\partial x_r} (\log N_a) + \frac{\partial S_{arr}}{\partial x_i} + S_{arr} \frac{\partial}{\partial x_i} \left( \log N_a \frac{K T_{aa}}{m_a} \right) \right] \\
+ \left( \frac{K T_{aa}}{m_a} \right)^2 \left[ 12 \frac{\partial u_{aar}}{\partial x_r} \right] \\
+ 12 u_{aar} \frac{\partial}{\partial x_r} \left( \log N_a \left( \frac{K T_{aa}}{m_a} \right)^2 \right) + 6 u_{aai} \frac{\partial}{\partial x_i} \left( \log \frac{m_a}{K T_{aa}} \right) \right]

\[ + 4 \left( \frac{e_a}{m_a} E_r + \frac{1}{m_a} F_{ar} \right) S_{arr} \]

\[ + \sum_{\sigma} \frac{N_{d_\sigma}}{\delta \sigma \delta_a} \left[ u_{\delta ar}^4 + 6 K T_{\delta a} u_{\delta ar}^2 + 3 \left( \frac{K T_{\delta a}}{m_a} \right)^2 \right], \]

\[ B_{arr} = - \left( \frac{K T_{aa}}{m_a} \right) \left[ \frac{\partial S_{arr}}{\partial x_s} + S_{arr} \frac{\partial}{\partial x_s} \left( \log N_a \frac{K T_{aa}}{m_a} \right) \right] \\
+ 3 \frac{\partial S_{arr}}{\partial x_r} + 3 S_{arr} \frac{\partial}{\partial x_r} \left( \log N_a \frac{K T_{aa}}{m_a} \right) \right]

\[ + 3 \frac{\partial S_{arr}}{\partial x_i} + 3 S_{arr} \frac{\partial}{\partial x_i} \left( \log N_a \frac{K T_{aa}}{m_a} \right) \right]

\[ + 3 \left( \frac{K T_{aa}}{m_a} \right)^2 \left[ \frac{\partial u_{aar}}{\partial x_s} + 2 u_{aar} \frac{\partial}{\partial x_s} \left( \log N_a \left( \frac{K T_{aa}}{m_a} \right)^2 \right) \right] \]

\[ + \frac{\partial u_{aar}}{\partial x_r} + u_{aar} \frac{\partial}{\partial x_r} \left( \log N_a \left( \frac{K T_{aa}}{m_a} \right)^2 \right) \right] \]
\[ C_{arrs} = - \left( \frac{K T_{aa}}{m_a} \right) \left[ 2 \frac{\partial S_{arrs}}{\partial x_r} + 2 S_{arrs} \frac{\partial}{\partial x_r} \left( \log N_a \frac{K T_{aa}}{m_a} \right) \right] \]

\[ + \frac{\partial S_{arrs}}{\partial x_s} + S_{arrs} \frac{\partial}{\partial x_s} \left( \log N_a \frac{K T_{aa}}{m_a} \right) \]

\[ + \frac{\partial S_{arrs}}{\partial x_t} + S_{arrs} \frac{\partial}{\partial x_t} \left( \log N_a \frac{K T_{aa}}{m_a} \right) \]

\[ + \frac{\partial S_{arrs}}{\partial x_i} + S_{arrs} \frac{\partial}{\partial x_i} \left( \log N_a \frac{K T_{aa}}{m_a} \right) \]

\[ + \left( \frac{K T_{ad}}{m_a} \right)^2 \left[ \frac{\partial u_{aat}}{\partial x_s} + u_{aat} \frac{\partial}{\partial x_s} \left( \log N_a \left( \frac{K T_{aa}}{m_a} \right)^2 \right) \right] \]

\[ + \frac{\partial u_{aas}}{\partial x_t} + u_{aas} \frac{\partial}{\partial x_t} \left( \log N_a \left( \frac{K T_{aa}}{m_a} \right)^2 \right) \]

\[ + 2 \left( \frac{e_a}{m_a} E_r + \frac{1}{m_a} F_{ar} \right) S_{arsl} + \left( \frac{e_a}{m_a} E_s + \frac{1}{m_a} F_{as} \right) S_{arrr} \]

\[ + \left( \frac{e_a}{m_a} E_t + \frac{1}{m_a} F_{at} \right) S_{arrr} \]

\[ + \sum_{\delta} N_\delta \left[ u_{aas} \frac{\partial}{\partial x_s} + u_{aas} \frac{\partial}{\partial x_t} \right] \]

\[ D_{arrs} = - \left( \frac{K T_{aa}}{m_a} \right) \left[ 2 \frac{\partial S_{assr}}{\partial x_r} + 2 S_{assr} \frac{\partial}{\partial x_r} \left( \log N_a \frac{K T_{aa}}{m_a} \right) \right] \]

\[ + 2 \frac{\partial S_{arrs}}{\partial x_s} + 2 S_{arrs} \frac{\partial}{\partial x_s} \left( \log N_a \frac{K T_{aa}}{m_a} \right) \]

\[ + \left( \frac{e_a}{m_a} E_r + \frac{1}{m_a} F_{ar} \right) S_{arsl} + \left( \frac{e_a}{m_a} E_s + \frac{1}{m_a} F_{as} \right) S_{arrr} \]

\[ + \left( \frac{e_a}{m_a} E_t + \frac{1}{m_a} F_{at} \right) S_{arrr} \]

\[ + \sum_{\delta} N_\delta \left[ u_{aas} \frac{\partial}{\partial x_s} + u_{aas} \frac{\partial}{\partial x_t} \right] \]
Transport processes in a multi-component assembly

+ $\frac{\partial S_{air}}{\partial x_i} + S_{air} \frac{\partial}{\partial x_i} \left( \log N_a \frac{K T_{aa}}{m_a} \right)$

+ $\frac{\partial S_{aix}}{\partial x_i} + S_{aix} \frac{\partial}{\partial x_i} \left( \log N_a \frac{K T_{aa}}{m_a} \right)$

+ $\left( \frac{K T_{aa}}{m_a} \right)^2 \left[ 2 \frac{\partial u_{aax}}{\partial x_r} + 2 u_{aax} \frac{\partial}{\partial x_r} \left( \log N_a \frac{K T_{aa}}{m_a} \right)^2 \right]$

+ $2 \frac{\partial u_{aas}}{\partial x_s} + 2 u_{aas} \frac{\partial}{\partial x_s} \left( \log N_a \frac{K T_{aa}}{m_a} \right)^2$

+ $2 u_{aai} \frac{\partial}{\partial x_i} \left( \log \frac{K T_{aa}}{m_a} \right)$

+ $2 \left( \frac{e_a}{m_a} E_r + \frac{1}{m_a} F_{ar} \right) S_{aarr} + 2 \left( \frac{e_a}{m_a} E_s + \frac{1}{m_a} F_{as} \right) S_{aarr}$

+ $\sum \frac{N_\delta}{\delta} \left[ u_{\delta ar}^2 + u_{\delta as}^2 + \frac{K T_{\delta a}}{m_a} \left( u_{\delta ar}^2 + u_{\delta as}^2 \right) \right]$

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